Stage 10 Knowledge Organiser

Non-Negotiables

Corbettmaths video numbers indicated in brackets

1) Factorise and Solve Quadratics (including of the form ax2 + bx + c =0 and rearranging) (266)
2) Recall and apply the Quadratic Formula (267)
3) Draw and interpret probability trees and calculate the probabilities of multiple events (252)
4) Draw and interpret histograms including calculating group width frequency density (157, 158, 159)
5) Recall and apply SOH CAH TOA trigonometry rules to find missing sides and angles in a triangle (329, 330, 331)
6) Recall and apply Pythagorean and Trigonometric rules to find missing sides and angles in 3D Shapes (259 for Pythagoras, 322 for Trigonomentry)
7) Generate and find the nth term of a quadratic sequence (388)
8) Apply negative and fractional index rules (173 and 175)
9) Calculate surds including adding, subtracting, multiplying (including expanding brackets) and dividing (305 and 306)
10) Rationalise the denominator of a surd (307)
11) Solve inequalities including describing and shading regions (182)
12) Find the volume and surface area of 3D shapes including sphere, pyramids and cones (355 – 361 dependent on shape)
13) Solve simultaneous equations including by multiplying (295 and 298)
14) Recall and apply the seven circle theorems (64 and 65)
15) Change a recurring decimal to a fraction (96)
16) Carry out transformations including reflections, rotations, translations and enlargements (reflections 272-274; rotations 275; translations 326; enlargements 104-106)
17) Find the equation of a circle and draw a circle using the equation (12)
18) Use vectors to solve geometrical problems (353)
19) Calculate average and instantaneous rates of change (390)
20) Identify roots, intercepts and turning points of quadratics graphically (267c)

<table>
<tr><th>Skill</th><th>Method</th><th>Keywords and Definitions</th></tr>
<tr>
<td>10-01</td>
<td>Factorise and solve quadratics (including of the form $ax^2 + bx + c = 0$ and rearranging)

Draw a table and separate your three coefficients, find the factor pairs for ax^2 and c which cross multiply to give you bx.

$2x^2+13x+15=0$

<table>
<tr><th>$2x^2$</th><th>$+13x$</th><th>$+15$</th></tr>
<tr><td>$2x$</td><td>$10x$</td><td>$+3$</td></tr>
<tr><td>x</td><td>$3x$</td><td>$+5$</td></tr>
<tr><td>$2x^2$</td><td>$+13x$</td><td>$+15$</td></tr>
</table>

Read across to find the two factorised brackets so:

$2x^2 +13x + 15 = 0$ factorises to $(2x + 3)(x + 5) = 0$

So that the two brackets multiply to give an answer of zero, one of our brackets must equal zero.
So $2x + 3 = 0$ or $x + 5 = 0$
Solve each equation, this is how we derive our two solutions.

So $2x + 3 = 0$ or $x + 5 = 0$
$2x = -3$ $x = -5$
$X = -\frac{3}{2}$</td>
<td>Quadratic Equation – an equation with an x^2 term

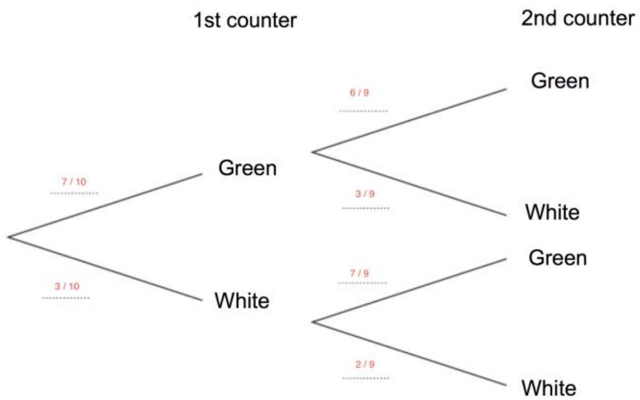
Coefficient – the number in front of a letter which multiplies is eg. 2x is 2 times x – the coefficient is the 2

Factorise- put into brackets</td>
</tr>
</table>

10-02	Recall the quadratic formula as $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ A useful video for remembering the formula can be found here: https://www.youtube.com/watch?v=-gwz6d9NYz0#action=share This applies to quadratics when in the form $ax^2 + bx + c = 0$ and is used when the equation cannot be factorised. Note: if your equation does not equal zero then you must rearrange it first For example: $2x^2 + 5x = 6$ rearranges to $2x^2 + 5x - 6 = 0$ Using this example our coefficients are a = 2, b = 5, c = -6 Substitute these values into the formula so: $$x = \frac{-(5) \pm \sqrt{(5)^2 - 4(2)(-6)}}{2(2)}$$ Which gives: $\frac{-5 \pm \sqrt{73}}{4}$ So: $\frac{-5+\sqrt{73}}{4} = 0.89$ or $\frac{-5-\sqrt{73}}{4} = -3.39$ x = 0.89 or x=-3.39 Note: these answers have been rounded to two decimal places	Substitute – replace the letter with the known value

<table>
<tr>
<td>10-03</td>
<td>Construct a probability tree diagram for the question.

There are 10 counters in a bag, 7 are green and the rest of white. Erin takes out a counter at random and records its colour. Without replacement, Erin takes out another counter, at random.

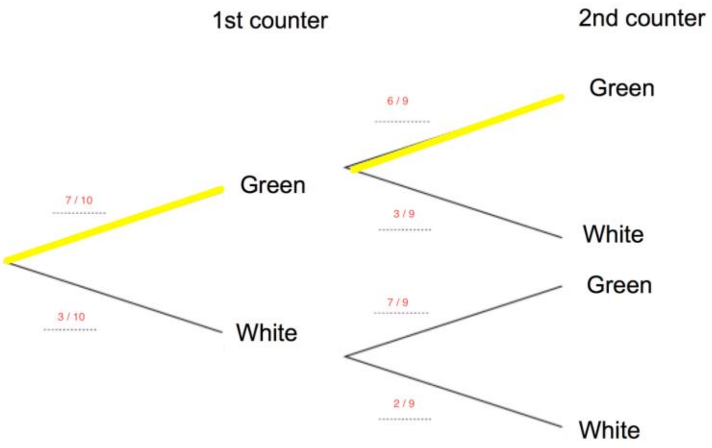


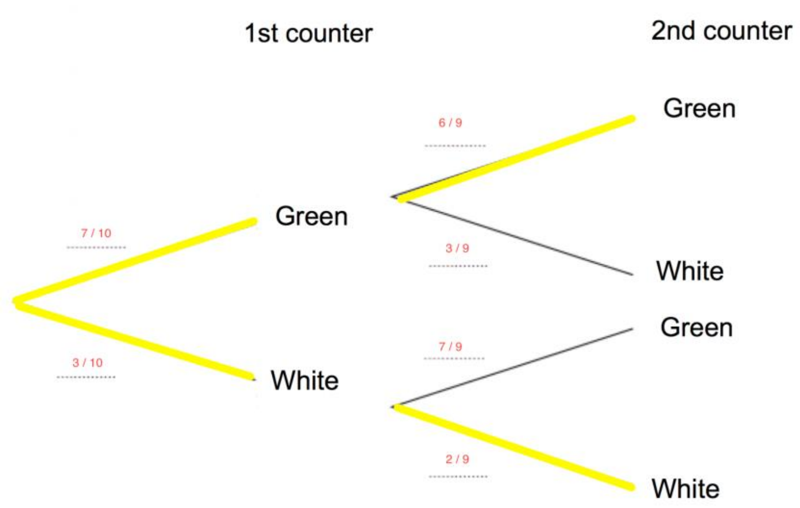
Note: Be careful with dependent and independent events – in this example Erin does not replace the counters and so the probability for the second counter depends on which colour was picked the first time
For example: there are 7 green counters and 3 white counters at the start, if Erin picks a green counter on her first pick this leaves 6 green and 3 white counters for the second pick however if she picks a white counter first then this would leave 7 green counters and 2 white counters.

If Erin replaced the counter after each pick then she would always have 7 green counters and 3 white counters so the probability would not change – this would be an example of independent probability.</td>
<td>Probability- how likely something is to happen

Independent event – the probability is not affected by other events

Dependent event – the probability changes dependent on other events in the tree diagram, also known as conditional probability</td>
</tr>
</table>

	To calculate the probability of multiple events multiply across the probability tree. For example: Calculate the probability that Erin will pick two green counters out of the bag.  Multiply the two probabilities, so the probability of choosing a green counter first ($\frac{7}{10}$) multiplied by the probability of choosing a green counter second ($\frac{6}{9}$) $\frac{7}{10} \times \frac{6}{9} = \frac{42}{90}$ So the probability of choosing two green counters is $\frac{42}{90}$ or $\frac{7}{15}$ simplified	

	You may also be asked to find the probability of more than one combination eg. Find the probability that Erin will pick two counters the same colour In this example you firstly should identify which combinations you need to calculate, then add them together.  Calculate the probability of each combination The probability of choosing two green counters is $\frac{7}{10}$ x $\frac{6}{9}$ $=\frac{42}{90}$ The probability of choosing two white counters is $\frac{3}{10}$ x $\frac{2}{9}$ $=\frac{6}{90}$ Then add them The probability of choosing two counters the same colour is $\frac{42}{90} + \frac{6}{90} = \frac{48}{90}$ ($\frac{8}{15}$ in simplest form)	

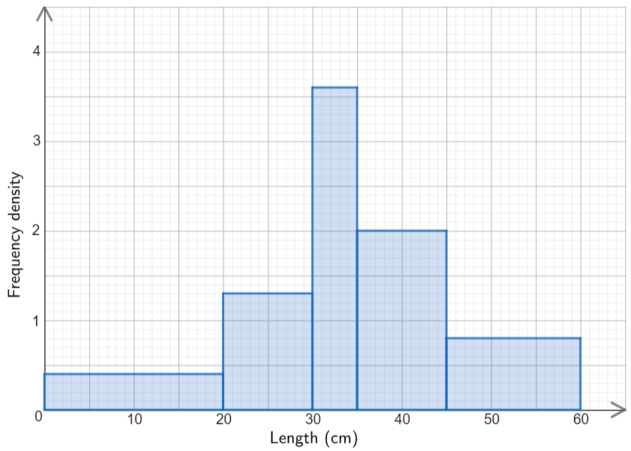
10 -04

Histograms have several key distinctions from a bar chart.
They always follow the same format.
The x axis shows the group width (this means if the groups are different sizes then the bars will be different widths)
The y axis shows the frequency density ($Frequency\ Density = \frac{Frequency}{Group\ Width}$)

To draw a histogram you first need to calculate the Frequency Density.

Length, l (cm)	Frequency	Frequency density
$0 < l \leq 20$	8	0.4
$20 < l \leq 30$	13	1.3
$30 < l \leq 35$	18	3.6
$35 < l \leq 45$	20	2
$45 < l \leq 60$	12	0.8

You can then plot this on to a histogram. Note the different width of the bars due to the different group widths, and the frequency density plotted on the y axis.



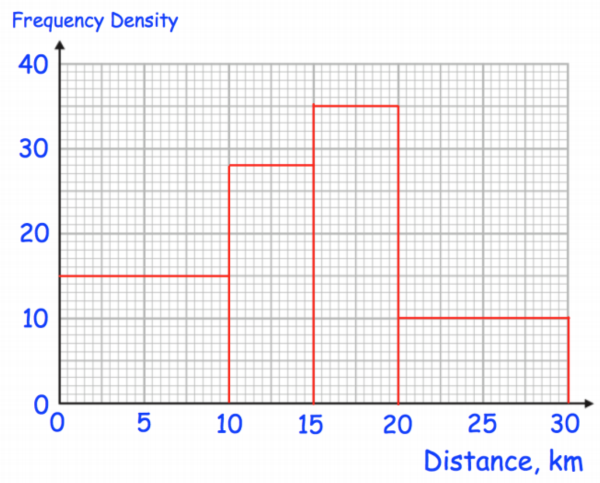
Group width – also known as class size eg. if measuring from 0-20 the group width is 20, if measuring from 21-30 the group width is 10 (not 9).

Frequency – how many times something happens in the sample

Frequency density – the frequency per unit, how many are in the group compared to the group width

You may alternatively be given a histogram and asked to interpret it. You can use your knowledge of histograms to do this.

Eg. The below histogram shows how far away a group of students live from school. Calculate how many students live less than 10km from the school.



We know $Frequency\ Density = \frac{Frequency}{Group\ Width}$

Therefore to find how many students live less than 10km from school (how many = frequency)
Rearrange the formula

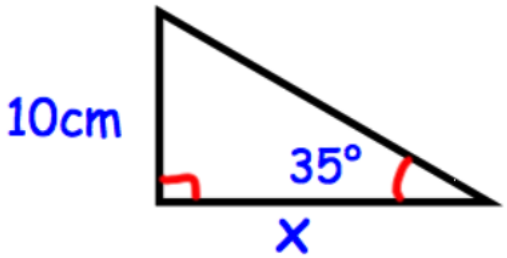
$$Frequency = Frequency\ Density\ x\ Group\ Width$$

Group Width = 10
Frequency Density = 15

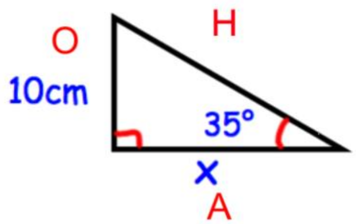
$$Frequency = 15\ x\ 10 = 150 \text{ students}$$

A useful way to remember this is frequency = area of the bar

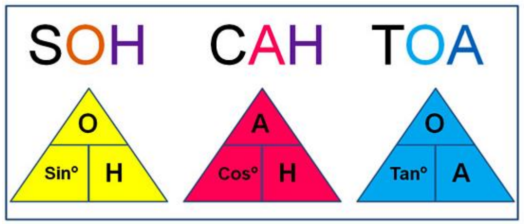
<table>
<tr>
<td>10 -5</td>
<td>
When using trigonometry to find the missing side or angle of a right angled triangle you firstly need to label your triangle.



Hypotenuse is the longest side (always facing the right angle)
Opposite is opposite the angle
Adjacent is beside the angle (always between your angle and the right angle



Now you can use SOH CAH TOA to calculate the missing side



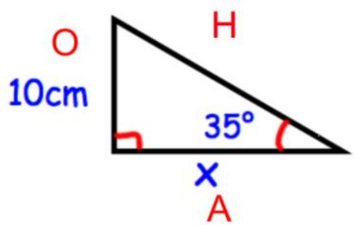
</td>
<td>
Hypotenuse is the longest side (always facing the right angle)

Opposite is opposite the angle

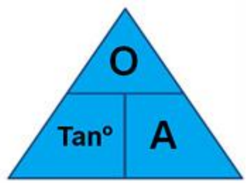
Adjacent is beside the angle (always between your angle and the right angle
</td>
</tr>
</table>

To use SOH CAH TOA firstly identify the two sides you are using (one you will know and the other is the one you are trying to find.

In this example, we know the length of O (10cm) and we are trying to find the length of A (x cm)



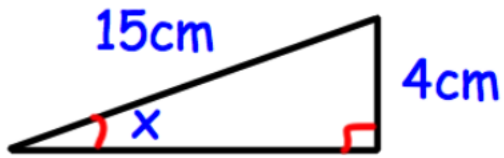
This means we will use TOA as it has A and O.



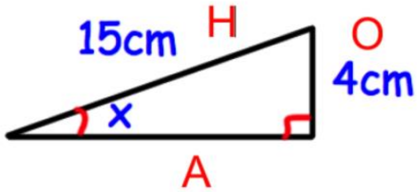
Cover the value you are trying to find and calculate:

Adjacent $= \frac{opposite}{Tan\ \theta} = \frac{10}{Tan\ (35)} =$ 14.28cm (rounded to 2dp)

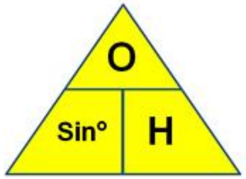
To find a missing angle the first steps are the same.



Label the sides



Identify the sides you are working with and choose your function.
In this case we know O (4cm) and H (15cm). Therefore we will use SOH.



$$Sin\ \theta = \frac{opposite}{hypotenuse} = \frac{4}{15}$$

θ - pronounced theta, is the symbol representing an unknown angle

<table>
<tr>
<td></td>
<td>As we are finding an angle, we need to use the inverse function of Sin.

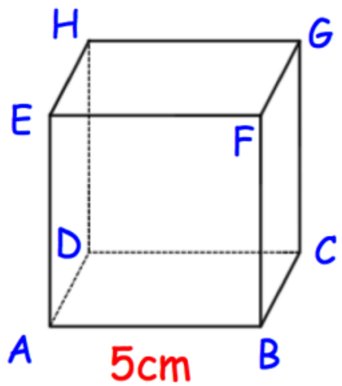
We know that

$Sin\ \theta = \frac{4}{15}$ so using the inverse we know

$\sin^{-1}\left(\frac{4}{15}\right) = \theta$

$\theta = 15.47°$ (rounded to 2dp)</td>
<td>Inverse is the reverse function and is used to find an angle – it is represented by the -1 notation</td>
</tr>
<tr>
<td>10-06</td>
<td>The same Pythagorean and Trigonometric rules you have used on triangles apply to 3D also. The main issue to focus on is finding the triangle within the shape.

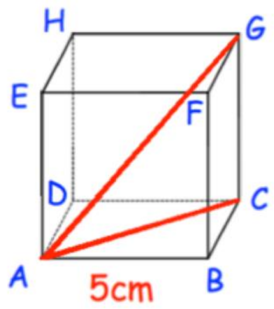
Find the length of AG in this cube



In this cube, the line AG is a diagonal line.

It is a hypotenuse of a right-angled triangle formed by the lines AC and GC.

See the diagram below.</td>
<td></td>
</tr>
</table>

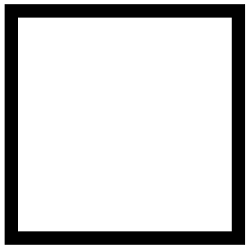


Therefore we can use Pythagoras' Theorem to calculate the length of AG

$a^2 = b^2 + c^2$

To find the length of AC consider the base of the cube

D 5cm **C**



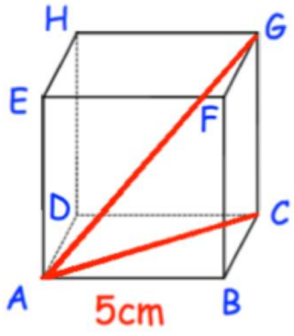
5cm

A 5cm **B**

To calculate AC, use Pythagoras

$a^2 = b^2 + c^2$
$a^2 = 5^2 + 5^2$
$a^2 = 50$

$a = \sqrt{50} = 7.07\text{cm}$ (rounded to 2dp)



So, AC = $\sqrt{50}$ cm
GC = 5cm

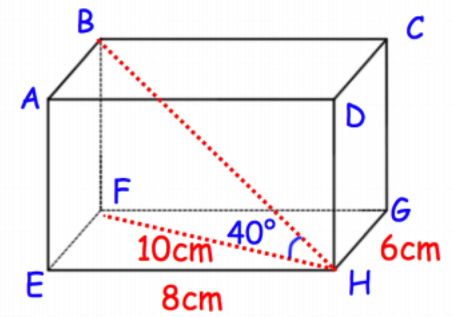
Now use Pythagoras to calculate the length of AG

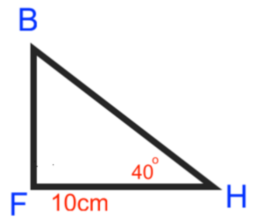
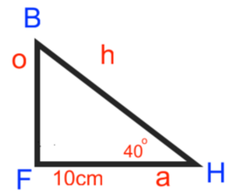
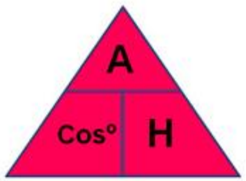
$a^2 = b^2 + c^2$
$a^2 = (\sqrt{50})^2 + (5)^2$
$a^2 = 50 + 25 = 75$
$a^2 = 75$
$a = \sqrt{75} = 8.66$cm (rounded to 2dp)

Note: you may use the decimal value for AC (7.07cm) however it is more accurate if you do not round until the last step if possible.

The same principles apply for using trigonometry in a 3D Shape. See the worked example below.

Calculate the length of BH.



<table>
<tr>
<td></td>
<td>Sketch the triangle BFH. Using trigonometry label the sides and identify the function


Therefore to find BH.

$h = \frac{adjacent}{\cos\theta} = \frac{10}{\cos(40)} = 13.05cm$ (rounded to 2dp)</td>
<td></td>
</tr>
<tr>
<td>10-07</td>
<td>To generate a sequence from a quadratic nth term remember that n represents the position in the sequence. So for the first term n=1, for the second n=2, etc.
Substitute the n value into the equation to find the corresponding value.

Generate the first five terms of the sequence $n^2 - 2n + 5$

1st term n=1 $(1)^2 - 2(1) + 5 = 4$
2nd term n = 2 $(2)^2 - 2(2) + 5 = 5$
3rd term n = 3 $(3)^2 - 2(3) + 5 = 8$
4th term n= 4 $(4)^2 - 2(4) + 5 = 13$
5th term n = 5 $(5)^2 - 2(5) + 5 = 20$

4, 5, 8, 13, 20</td>
<td>Nth term – the term representing the value of a sequence</td>
</tr>
</table>

To calculate the nth term from a quadratic sequence, first find the n^2 term and subtract it from the sequence. You'll then be left with a linear nth term to calculate.

Find the nth term of the sequence 7, 12, 19, 28, 39 …

Note: we know this is a quadratic sequence as the sequence increases by a different amount each time.

The sequence increases +5, +7, +9, +11

The second difference is +2 as this is the amount increased each time.

This gives a quadratic term of n^2.

Note the n^2 term is half the second difference eg. a second difference of 4 = $2n^2$, a second difference of 8 = $4n^2$.

So, our sequence has a term n^2

Sequence	7	12	19	28	39
n	1	2	3	4	5
n^2	1	4	9	16	25

Subtract the n^2 term from the sequence

Sequence	7	12	19	28	39
n^2	1	4	9	16	25
	6	8	10	12	14

We are left with the linear sequence 6, 8, 10, 12, 14

Which has an nth term 2n + 4 (common difference of 2, +4 to get first term)

Therefore our sequence has the nth term of $n^2 + 2n + 4$

Note: the nth term of $n^2 + 2n + 4$, if we subtract n^2 from this we are left with 2n + 4

This allows you to subtract the n^2 term and find the nth term of the remaining sequence

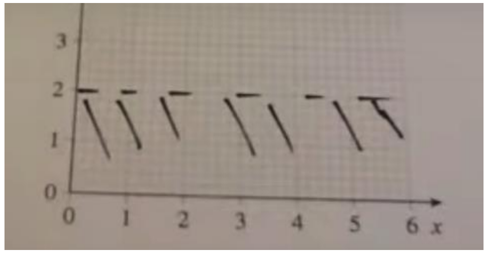
10-08	There are several important rules to remember regarding negative and fractional indices. The denominator of the fraction is the root of the number Eg. $25^{\frac{1}{2}}$ means the second root, or <u>square root</u>, of the number so $25^{\frac{1}{2}}$ means $\sqrt{25} = 5$ $81^{\frac{1}{2}} = \sqrt{81} = 9$ $64^{\frac{1}{3}} = \sqrt[3]{64} = 4$ If you have a numerator greater than one, then you must raise your answer to than power Eg. $9^{\frac{3}{2}} = (\sqrt{9})^3 = 3^3 = 27$ $64^{\frac{2}{3}} = (\sqrt[3]{64})^2 = 4^2 = 16$ A negative power can be referred to as a reciprocal. In practice this means 1 divided by the power. If you consider using index laws of division $x^4 \div x^5 = x^{-1}$ This represented another way is $\frac{y \times y \times y \times y}{y \times y \times y \times y \times y} = \frac{1}{y}$ Therefore $y^{-2} = \frac{1}{y^2}$ These two rules on negative and fractional indices can be combined For example $y^{-\frac{3}{2}} = \frac{1}{y^{\frac{3}{2}}}$ and $9^{-\frac{3}{2}} = \frac{1}{9^{\frac{3}{2}}} = \frac{1}{(\sqrt{9})^3} = \frac{1}{3^3} = \frac{1}{27}$	Reciprocal – 1 divided by the number eg reciprocal of 2 is $\frac{1}{2}$, reciprocal of x is $\frac{1}{x}$

10-09	When multiplying surds remember the rule $\sqrt{a}\ x\ \sqrt{b} = \sqrt{ab}$ eg. $\sqrt{2}\ x\ \sqrt{8} = \sqrt{16} = 4$ The same is true for dividing $\sqrt{a} \div \sqrt{b} = \sqrt{a \div b}$ eg. $\sqrt{15} \div \sqrt{5} = \sqrt{3}$ Note: if you can simplify your surd as in the first example then you should, otherwise you should leave it as a surd, as seen in the second example. If your surd has a coefficient, these should be multiplied or divided separately Eg. $2\sqrt{3}\ x\ 5\sqrt{2} = 10\sqrt{6}$ $6\sqrt{15} \div 2\sqrt{3} = 3\sqrt{5}$ You can add or subtract surds if the number under the square root sign is the same. Eg. $5\sqrt{3} + 3\sqrt{3} = 8\sqrt{3}$ $12\sqrt{5} - 10\sqrt{5} = 2\sqrt{5}$ If the number under the square root sign is different then you can't add or subtract (not like terms)	A surd is an irrational square root eg. $\sqrt{5}$ is a surd, but $\sqrt{4}$ is not as $\sqrt{4} = 2$

10-10	Rationalising a denominator means to simplify a fraction so the denominator is not a surd. To rationalise a denominator multiply by the surd on the denominator over itself Eg. To rationalise $\frac{5}{\sqrt{3}}$ multiply by $\frac{\sqrt{3}}{\sqrt{3}}$ this will eliminate the surd on the denominator, without changing the value of the fraction. This does not change as any number divided by itself equals 1. So when we multiply by $\frac{\sqrt{3}}{\sqrt{3}}$ we are actually multiplying by 1. Eg. Rationalise $\frac{5}{\sqrt{3}}$ $\frac{5}{\sqrt{3}} \; x \; \frac{\sqrt{3}}{\sqrt{3}} = \frac{5 \; x \; \sqrt{3}}{\sqrt{3} \; x \; \sqrt{3}} = \frac{5\sqrt{3}}{\sqrt{9}} = \frac{5\sqrt{3}}{3}$	
10-11	An inequality solves in exactly the same manner as an equation Eg. 3n < 9 divide both sides by 3 n < 3 2n – 5 > 12 add 5 to both sides 2n > 17 divide both sides by 2 n > 8.5 Note: a common mistake people make is inserting an equals sign instead of an inequality sign, out of habit. Make sure you keep the same inequality sign throughout.	Inequality - two expressions that are not equal eg. x < 2 means x is less than two

When asked to shade regions first solve the inequality in terms of y or x, then shade the area of the graph which does not meet the criteria of your inequality.

For the inequality y > 2 (y is greater than 2), you would shade all of the graph where y is less than 2 as this does not meet the criteria (y cannot be 1 for example).

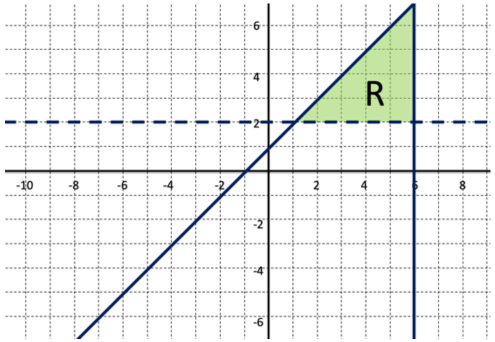
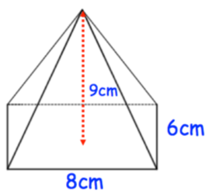


If you need to solve a more difficult equation such as y < 2x + 1, then create a table of values for x and y and solve to find co-ordinates for the inequality to plot on your graph.

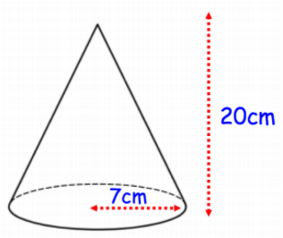
X	0	1	2
Y	1	3	5

You now have three co-ordinates to plot the inequality – (0,1), (1,3) and (2,5).

You will usually be given three inequalities to shade.
Solve each of them in turn and shade the corresponding area of the graph.
Once this is complete, the region which has <u>not been shaded</u> is the region which satisfies all of your inequalities, label this region R.

	Eg. Shade the region that satisfies the inequalities $y \leq x + 1$ $x \leq 6$ $y > 2$ Label the region R  Note: Inequalities that use < or > symbols are plotted with a dotted line to show that the line is not included in the region. Inequalities that use ≤ or ≥ symbols are plotted with a solid line to show that the line is included in the region.	
10-12	There are several formulae you need to be able to use to find the volume or surface area of a 3D shape. Although these are given to you in the exam, it is important you are confident identifying which formula to use, and how to apply it. Volume of a Pyramid or Cone = $\frac{1}{3}$ $x\ area\ of\ base\ x\ h$ (h = height) Remember for a cone the base is a circle, so the area is πr^2 Find the volume of the pyramid  v = $\frac{1}{3}$ $x\ area\ of\ base\ x\ h$ area of base = 6 x 8 = 48cm^2 $=$ $\frac{1}{3}$ x 48 x 9 = 144cm^3	Volume – total space taken up by a 3D shape

Find the volume of the cone



$v = \frac{1}{3}\ x\ area\ of\ base\ x\ h$

$v = \frac{1}{3}\ x\ 49\pi\ x\ 20$

$v = \frac{980\pi}{3}\ cm^3$

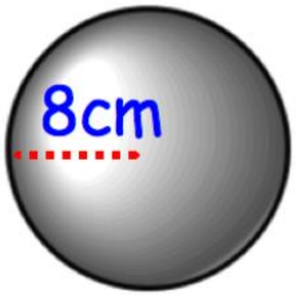
$v = 1026.25\ cm^3$ (rounded to 2dp)

area $= \pi r^2$

$= \pi\ x\ 7^2 = 49\pi\ cm^2$

Find the volume of the sphere

$$v = \frac{4}{3}\pi r^3$$



$v = \frac{4}{3}\pi r^3$

$v = \frac{4}{3}\ x\ \pi\ x\ 8^3$

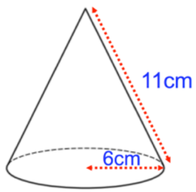
$v = \frac{2048}{3}\ \pi cm^3$

To find the surface area of a pyramid, simply find the area of all five sides (the four triangle faces as well as the base.

To find the surface area of a cone is slightly different. You need to find the area of the curved surface, as well as the circular base.

The formula for the curved surface area is $\pi\, x\; radius\; x\; length\; (of\; the\; slope)$

Find the surface area of the cone:



Curved SA = $\pi\, x\; radius\; x\; length\; (of\; the\; slope)$

= $\pi\, x\; 6\; x\; 11$

= 66π cm^2

Area of base = πr^2 = π x 6^2 = 36π cm^2

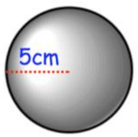
Then find the total surface area by adding the two together.

$36\pi + 66\pi = 102\pi$ cm^2 = 320.44 cm^2 (rounded to 2dp)

Note: this formula can be simplified to $\pi rl + \pi r^2$ if you prefer

The surface area of a sphere has a specific formula which is $4\pi r^2$

Find the surface area of the sphere



SA = $4\pi r^2$

= $4\; x\; \pi$ x 5^2

= 100π cm^2

= 314.16 cm^2 (rounded to 2dp)

Surface area – the area of each of the faces which form a 3D shape

10-13	To solve simultaneous equations, first eliminate one of the variables and solve for the equation. You can then use this to substitute into your equations to find the second variable. For example Solve the simultaneous equations $5x + y = 15$ $3x + y = 11$ the y values have the same coefficient so I can subtract the equations to eliminate y $\begin{array}{r} 5x + y = 15 \\ \underline{-\ 3x + y = 11} \\ 2x \quad = 4 \\ x = 2 \end{array}$ now I can solve for x Now that I have a value for x I can use this to find y by substituting the value into one of my equations. I know that $5x + y = 15$ so I can substitute $x = 2$ into this $5(2) + y = 15$ $10 + y = 15$ now solve for y $-10 \quad -10$ $y = 5$ So the solution is $x = 2$, $y = 5$	Simultaneous equations – two equations with each of the same unknowns which are solved together There is one value for each of the unknowns which makes both equations true

Sometimes the equations will not have any common coefficients which make it more difficult to eliminate a variable.

Eg. Solve the simultaneous equations

$3x + 4y = 4$
$2x + 3y = 2$ in this instance we have to multiply the equations to get a common coefficient
I will multiply my equations to get a common x coefficient

$3x + 4y = 4 \quad \times 2 \quad = \quad 6x + 8y = 8$
$2x + 3y = 2 \quad \times 3 \quad = \quad 6x + 9y = 6$ I can now subtract the equations and solve as before

$$\begin{array}{r} 6x + 9y = 6 \\ - \underline{6x + 8y = 8} \\ y = -2 \end{array}$$

(note I have switched equations around, this is so I keep a positive y coefficient)

Again, now I have a value for y I can solve for x by substituting into my original equation.

$3x + 4y = 4$
$3x + 4(-2) = 4$
$3x – 8 = 4$
$+8 \quad +8$
$3x = 12$
$\div 3 \quad \div 3$
$x = 4$

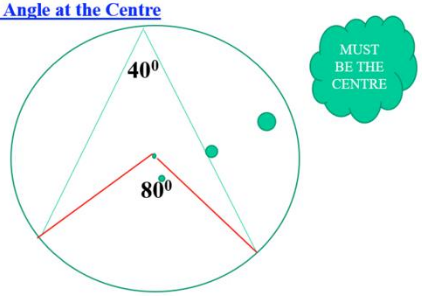
So my solutions are $x = 4$, $y = -2$

10-14

Circle Theorems

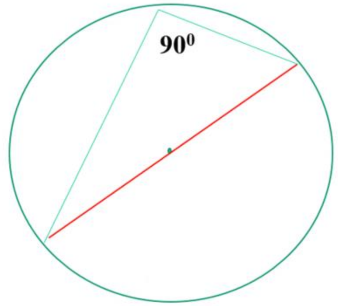
Circles have different angle properties, described by theorems. There are seven circle theorems.

Theorem 1: Angle at the Centre



THEOREM 1: ANGLE at the CENTRE of the CIRCLE is twice the angle at the circumference

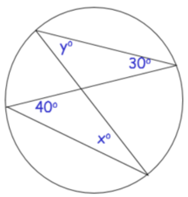
Theorem 2: Angles in a Semi-Circle



THEOREM 2: The angle at the circumference of a SEMICIRCLE is a right angle.

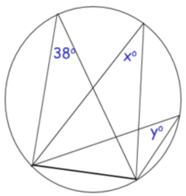
Theorem 3	Angles in the same segment are equal.

Find the unknown angles in each case



Angle x = 30°

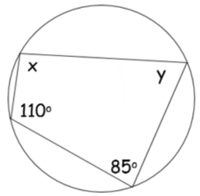
Angle y = 40°



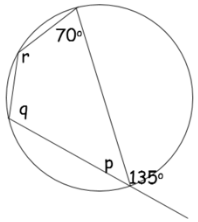
Angle x = angle y = 38°

Theorem 4	Cyclic Quadrilateral Theorem.

The opposite angles of a cyclic quadrilateral add to 180°

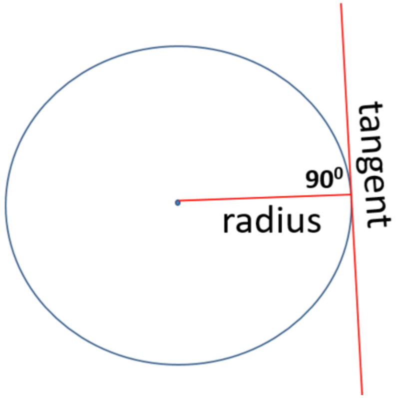
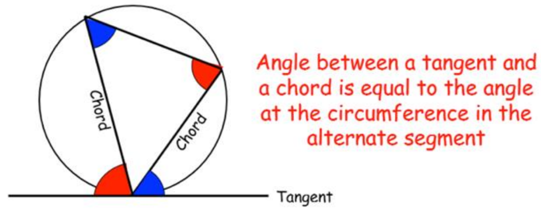
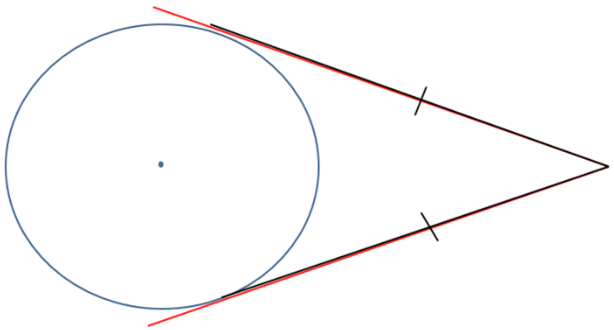
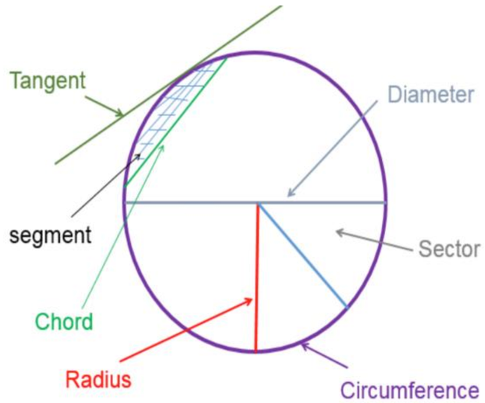


angle x =	180 – 85 = 95° (cyclic quad)
angle y =	180 – 110 = 70° (cyclic quad)

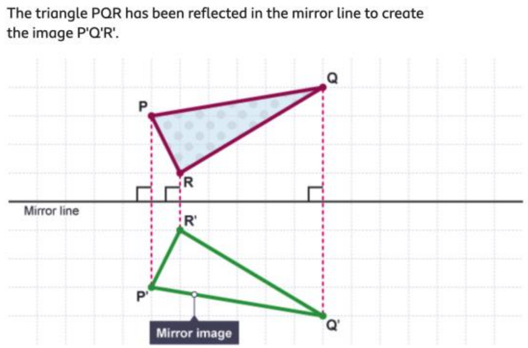
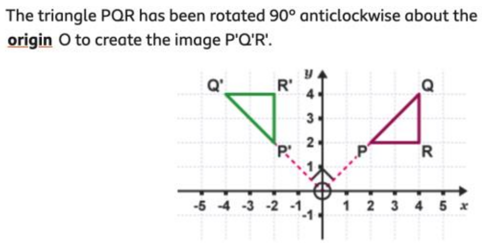


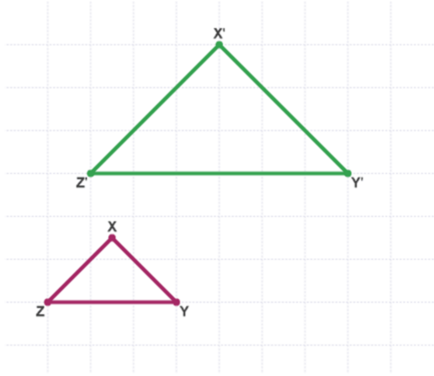
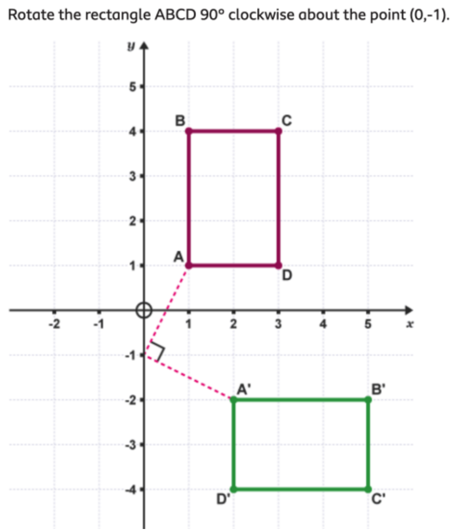
angle p =	180 – 135 = 45° (straight line)
angle q =	180 – 70 = 110° (cyclic quad)
angle r =	180 – 45 = 135° (cyclic quad)

See the circle diagram at the end of the topic

	Theorem 5 – A tangent is perpendicular to a radius  Theorem 6 - Alternate Segment Theorem  Theorem 7 – Tangents to a circle from the same point are equal in length  **<u>Diagram of circle</u>** 	

10-15	Change a recurring decimal to a fraction You can change a fraction to a decimal by division. Remember a fractions means the numerator divided by the denominator Eg. $\frac{1}{4}$ means 1 divided by 4 which is 0.25 $\quad 4\overline{)1.00}$ = 0.25 If you want to change a decimal to a fraction then consider place value 0.25 is 25 hundredths so therefore is $\frac{25}{100}$ which simplifies to $\frac{1}{4}$ This is slightly different for a recurring decimal as it is recurring and so it has no end point to use place value. We can solve this using an algebraic method to eliminate the recurring decimal. Write $0.\dot{7}$ as a fraction (note the accent above the 7 indicates it is recurring) Let $x = 0.\dot{7}$ so therefore $10x = 7.\dot{7}$ If we subtract x from 10x then $10x = 7.\dot{7}$ $-\quad x = 0.\dot{7}$ Therefore $9x = 7$ So $x = \frac{7}{9}$	Recurring – repeated and will not end

10-16	A transformation is a way of changing the size or position of a shape. Examples of transformations include reflection, rotation, translation or enlargement. A shape can be reflected across a line to create an image which is a mirror image of the original. The line of reflection is called the mirror line. (Mirror because this is where the reflection happens) Every point on the image is the same distance from the mirror line as the new image  A rotation turns a shape about a fixed point, called the centre of rotation. Three pieces of information are needed to rotate a shape: the centre of rotation, the angle of rotation and the direction of rotation 	Transformation - a way of changing the size or position of a shape. Image - the new shape created by carrying a transformation Reflection – creating a mirror image of a shape across a mirror line Rotation – turning a shape across the centre of rotation Centre of rotation – the point from which a shape is rotated

	An enlargement changes the size of a shape  The centre of enlargement may not always be the origin 	Centre of enlargement – the point from which a shape is enlargement

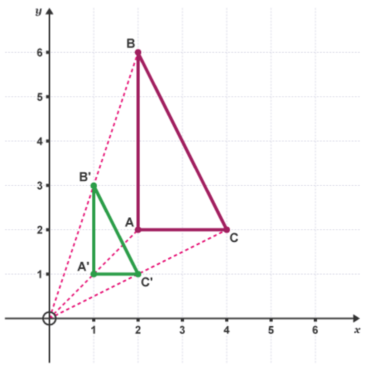
All the sides of the triangle X'Y'Z' are twice as long as the sides of the original triangle XYZ. The triangle XYZ has been enlarged by a scale factor of 2.

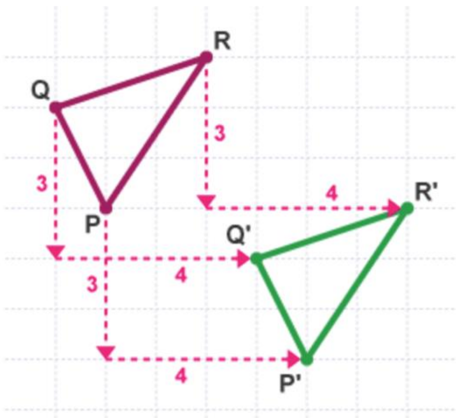
To enlarge a shape, a **centre of enlargement** is required. When a shape is enlarged from a centre of enlargement, the distances from the centre to each point are multiplied by the scale factor.

Note: an enlargement does not necessarily mean the shape gets bigger. For example a scale factor of $\frac{1}{2}$ will result in the new shape being half the size of the original

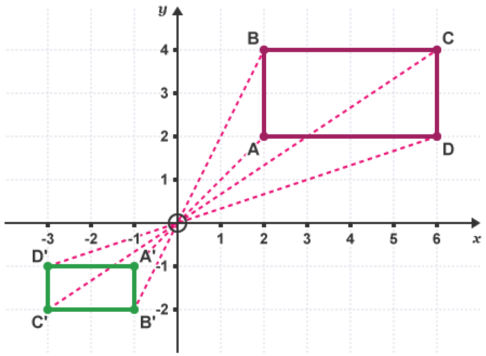
To enlarge a shape, a **centre of enlargement** is required. When a shape is enlarged from a centre of enlargement, the distances from the centre to each point are multiplied by the scale factor.

To find the centre of enlargement, draw ray lines from the corners of the image through the corners of the original shape. Your lines will all meet at one specific point. This is the centre or enlargement.



	An enlargement with a negative scale factor produces an image on the other side of the centre of enlargement. The image appears upside down. The rectangle ABCD has been enlarged by a scale factor of $-\frac{1}{2}$. The lengths in rectangle A'B'C'D' are $\frac{1}{2}$ times as long as rectangle ABCD. The distance from O to A'B'C'D' is half the distance from O to ABCD. 	

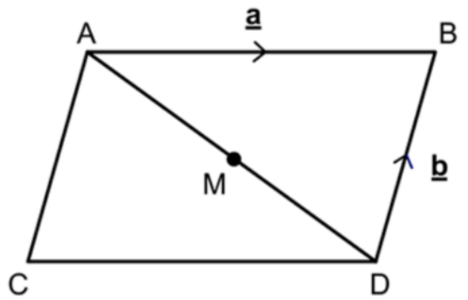
A translation moves a shape up, down or across but does not change anything else so the shape does not change size or rotate.
Every point of the original shape is translated the same distance and direction to create the new shape.



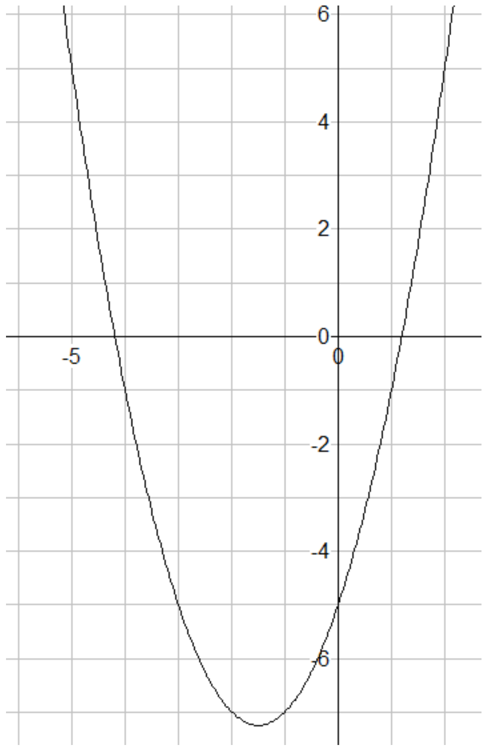
Column vectors are used to describe translations.

$\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ means translate the shape 4 squares to the right and 3 squares down.

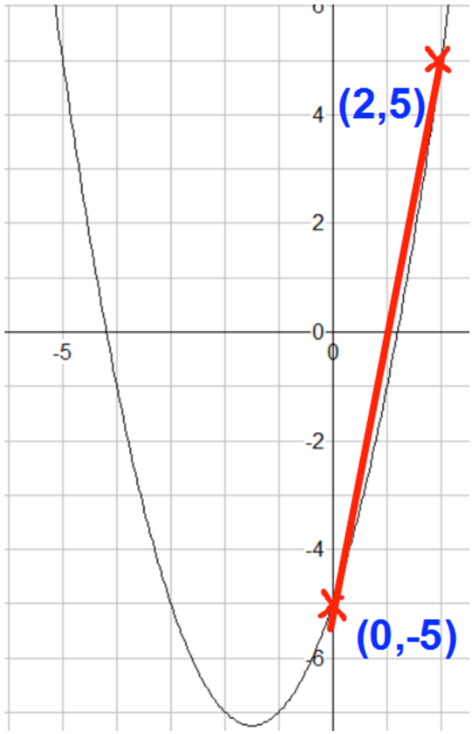
$\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ means translate the shape 2 squares to the left and 1 square up.

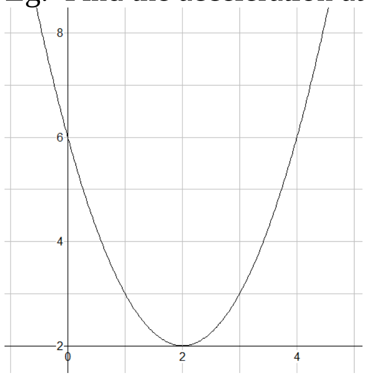
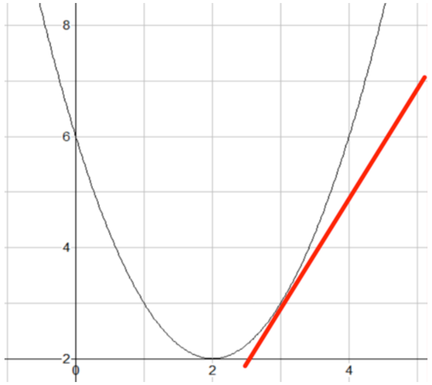
10-18	Use vectors to solve geometrical problems Look back over vectors on the stage 9 Knowledge Organiser before you attempt this The parallelogram ABCD is shown. M is the midpoint of AD.  Consider the magnitude and direction of the vectors. Eg. Find the vector $\overrightarrow{CM}$ in terms of a and b. So I do you get from C to M using vectors. Remember CD is parallel AB and so has a vector $\vec{a}$ And DB bas a vector $\vec{b}$ Therefore $\overrightarrow{CM} = \vec{a} + \vec{b}$ = a+b	Vector – a line which has both magnitude and direction

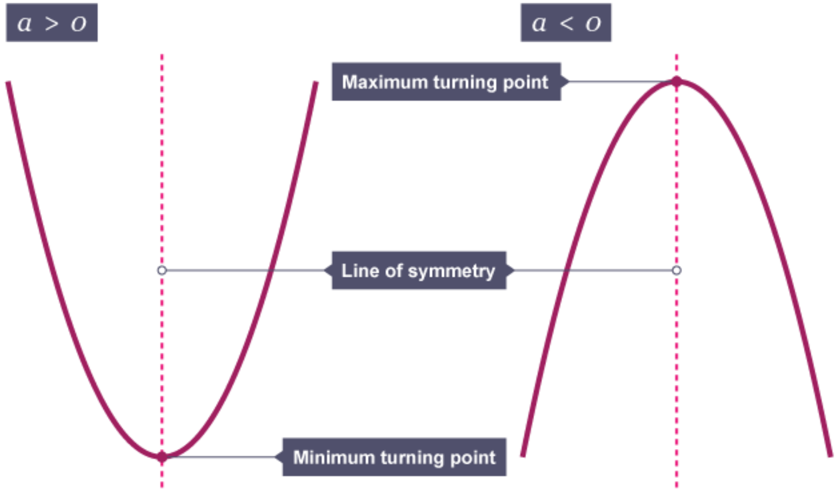
<table>
<tr>
<td>10-19</td>
<td>
A rate of change is the measure of the change in one variable compared to a change in another. For example speed is the rate of change in distance compared to the change in time. Acceleration is the rate of change in speed compared to the change in time. You can also compare the rate of change in price of something (a product, a share, a car etc) compared to time.
These variables that are compared can be modelled by graphs if we let one of them be shown on the x axis and one of them on the y axis. In this case the rate of change is the gradient of the graph.
In Maths there are two types of rates of change, average and instantaneous.

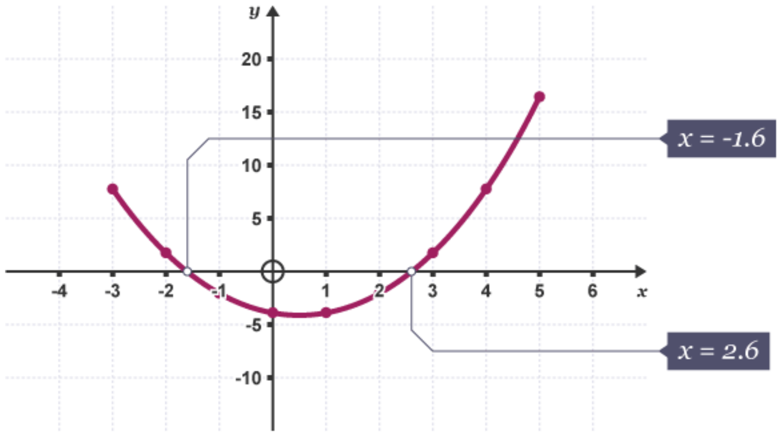


Here is the graph of
$y=x^2+3x-5$
What do you notice about the gradient of the curve?
The gradient changes.
So we cannot calculate the gradient in the same way as normal.
To calculate the average rate of change between two points we calculate the gradient of the cord between them
</td>
<td>Rate of Change – the change in one variable compared to another</td>
</tr>
</table>

	 So to find the average rate of change we need to find the gradient of the line Gradient = $\frac{change\ in\ y}{change\ in\ x}$ Gradient = $\frac{y2 - y1}{\mathrm{x2 - x1}} = \frac{5 - -5}{2 - 0} = \frac{10}{2} = 5$	Gradient – how steep the line is – a steeper line indicated a greater rate of change

	The instantaneous rate of change is the rate of change at a single point, not between two points on the curve. This is usually the acceleration if looking at a velocity time graph. Eg. Find the acceleration at the point x = 3  To do this, you need to draw a tangent to the curve at the point x = 3 and then find the gradient of the line  Now find the gradient of the line as before.	Instantaneous rate of change – the rate of change a specific point, measure by finding the gradient of the line at that point

10-20	All quadratic functions have the same type of curved graphs with a line of symmetry. The graph of the quadratic function $y = ax^2 + bx + c$ has a minimum turning point when $a > 0$ and a maximum turning point when a $a < 0$. The turning point lies on the line of symmetry.  	Turning point – the maximum or minimum point of the line

	The roots of an equation are the solution at the point where the line crosses the x axis  This can also be solved algebraically by using y = 0 If the equation has just one solution (a repeated root) then the graph just touches the x-axis without crossing it. If the equation has no solutions then the graph does not cross or touch the x-axis.	Roots – where the line crosses the x axis (y=0)