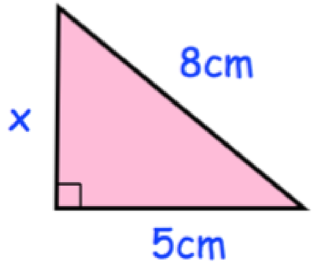
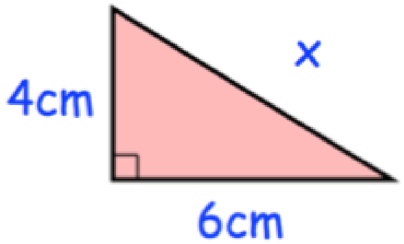
Stage 9 Knowledge Organiser (Corbett Maths video numbers in brackets)

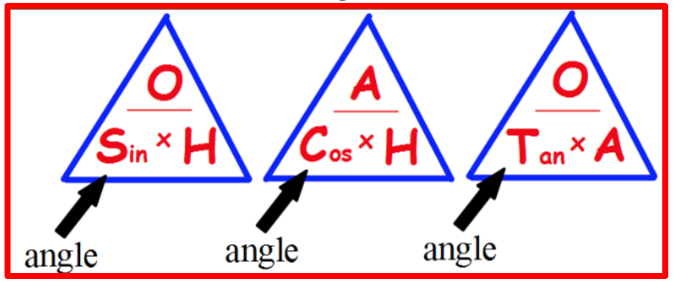
1. Calculate the value of an investment using compound interest (236)
2. Use Pythagoras theory to find any missing side on a right triangle (257)
3. Use SOH CAH TOA to find missing sides and angles on a right triangle (330, 331)
4. Expand double brackets (14)
5. Factorise double brackets with a coefficient of 1 for x^2 and use to solve a quadratic equation (118, 266)
6. Draw and use a tree diagram to find the probability of multiple events (252)
7. Find the equation of a straight line given two points (195)
8. Solve a pair of linear simultaneous equations (295)
9. Identify and understand positive and negative correlation (168)
10. Draw a line of best fit and use to estimate values on a scatter graph (167)
11. Plot a quadratic graph (264)
12. Solve an inequality and express on a number line (177, 178)
13. Solve a work done problem (254, 255)
14. Use similarity to find missing sides on shapes (292)
15. Calculate the volume of a sphere/pyramid/cone (359, 360, 361)
16. Identify and continue a Fibonacci sequence (287a)
17. Identify and generate/continue a quadratic sequence (388)
18. Know the rules for proving congruency (66, 67)
19. Add and multiply column vectors (353a)
20. Construct angle and line bisectors (72, 78)

Skill	Method	Keywords/Definitions
901	Calculating the Value of an Investment using Compound Interest Find the multiplier to increase by your rate of interest and multiply your original investment by the multiplier to the power of the number of years you are investing for. Example: Calculate the value of a £4000 investment at 1.2% interest for 5 years 1.2% interest gives us a multiplier of 1.012 (101.2%) over 5 years means to the power 5 £4000 x 1.012^5 = £4245.83	Compound interest is where you gain more interest each year on an investment as the amount in your account increases each year. Investment is when you put money into something.
902	Use Pythagoras to Find Missing Sides on a Right-Angled Triangle Pythagoras states: $\boxed{a^2 + b^2 = c^2}$ where c is the hypotenuse (longest side) TIP:If you are finding the biggest side you'll be adding, if you're finding a smaller side you'll be subtracting. Example: Find the missing sides on these triangles  The 8 is the biggest side here, so we are Finding a smaller side so we'll be subtracting: $x^2 + 5^2 = 8^2$ $x^2 + 25 = 64$ $x^2 = 39$ x =6.24cm  The x is the biggest side here so we'll be adding: $4^2 + 6^2 = x^2$ $16 + 36 = x^2$ $40 = x^2$ 6.32cm= x Check that the side you think is the biggest on the triangle is still the biggest side?	Hypotenuse-longest side on a right-angled triangle

903

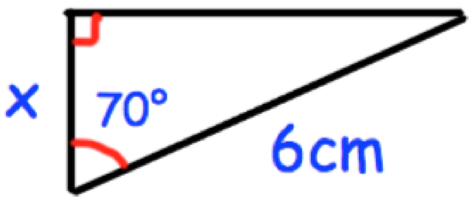
Use SOH CAH TOA (trigonometry) to Find Missing Sides and Angles on Right-Angled Triangles

Learn these formula triangles:



Label the sides of the triangle Opposite, Adjacent and Hypotenuse then whichever are the important sides (ones given and that you're working out) will direct you to one of the above triangles. Fill in what you know in the formula triangle and it will tell you what to put into your calculator.

Examples: Find the missing side x

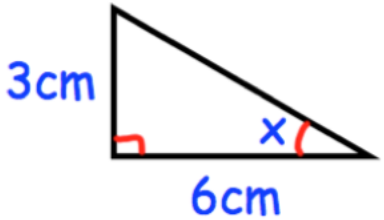


Labelling sides in respect to the 70° means
The x will be A (adjacent to the angle)
The 6cm is the H for Hypotenuse (longest side)
And the top side will be O (Opposite the angle)

The 6cm and the x are important here, so we are
Using A and H which means we use the Cosine triangle
Filling in what we know gives us

$\cos(70) \times 6 = 2.05cm$

Find the missing angle x



Labelling sides with respect the angle x means
The 3cm will be O (opposite the angle)
The 6cm will be A (adjacent to the angle)
The diagonal/blank side will be H (Hypotenuse)

The 3cm and the 6cm are the important sides, so we are using A and O which means using the Tan triangle. Filling in what we know gives

$\tan(x) = \frac{3}{6}$

To get x on its own here we need to do $\tan^{-1}$

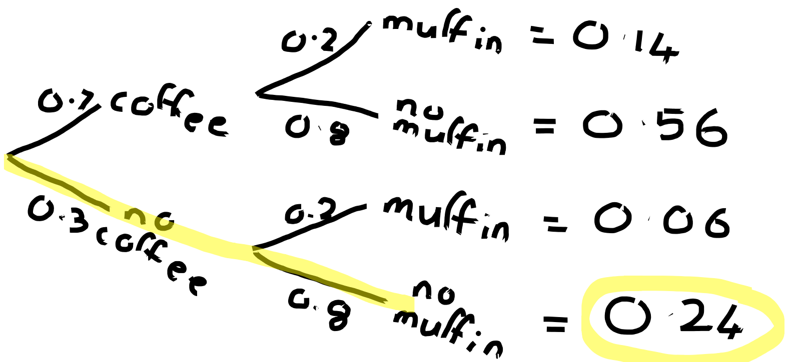
$\tan^{-1}\left(\frac{3}{6}\right) = 27^{o}$

Trigonometry – the study of triangles
Sin cos and tan are all trigonometric ratios.
Hypotenuse – biggest side on a right-angled triangle.
Opposite side – the side opposite the angle you're given in the question.
Adjacent side – the side 'next to' the angle given to you in the question.

When finding a missing angle, the final step will require you to do an inverse of either sin, cos or tan. These are known as $\sin^{-1}$ $\cos^{-1}$ and $\tan^{-1}$ (kind of like dividing by the ratios) these are done by pressing shift on the calculator following by the ratio you want to use.

904	Expand Double Brackets Multiply both terms in the first bracket by both terms in the second bracket. Use FOIL to remember this:(Firsts Outers Inners Lasts) if you need to. Example: Expand $(x+4)(x-6)$ Firsts: x^2 Outers: $-6x$ Inners: $+4x$ Lasts: -24 So, the initial expansion gives $x^2 - 6x + 4x - 24$ Which simplifies to $x^2 - 2x - 24$	Be really careful of negatives when expanding brackets like this. Only the two middle terms are like terms, remember an x term and an x^2 term are no alike.
905	Solve a Quadratic Equation by Factorising To factorise into two brackets we have to find the pair of factors of the last term that will add to give the middle term. Once we know the two brackets we use the inverse of each bracket to find our solutions to the equation. Example: Solve $x^2 - 3x - 10 = 0.$ Look for a factor pair of ten that can make -3: 10 1 × 10 2 × 5 If we use +2 and -5 this will give us the -10 at the end of the equation and simplify to give the -3x $(x+2)(x-3) = 0$ This is now factorised, but not solved. To solve it ask yourself how to make each bracket equal zero? $(x+2) = 0$ gives $x = -2$ $(x-5) = 0$ gives $x = +5$ So, the solutions are x=-2 and x=5	Factor pairs of a number multiply to give that number. Solutions to an equation are the missing numbers that the letter can take to make the equation work.
906	Draw and Use a Tree Diagram to Find Probabilities Tree diagrams are used when one thing happens after another. In a tree diagram each group of branches always adds up to 1. Multiply along the branches of the tree to find the combined probabilities, then find the one(s) that you are looking for, and add them up if there's more than one.	

<table>
<tr><td></td><td>Example:
The probability Sara has coffee on a morning is 0.7 and the probability she has a muffin is 0.2
Find the probability she has neither a coffee or a muffin



The probability of not having a coffee and not having a muffin is 0.24</td><td></td></tr>
<tr><td>907</td><td>Find the Equation of a Straight Line From Two Coordinate Points

Remember that the equation of a straight line can be represented as $y = mx + c$ where m is the gradient and c is the y intercept. We need to find both m and c using the two points we are given.

+12
Example: find the equation of the line passing through the two points (2 , 7) and (5 , 19)
+3

Find the gradient between the two points first: $\frac{change\ in\ y}{change\ in\ x} = \frac{+12}{+3} = 4$ so, we know the equation is $y = 4x + c$

Next sub one of the points (x,y) into your equation and solve to find c

Putting (2,7) into $y = 4x + c$ gives:

$7 = 4(2) + c$
$7 = 8 + c.$ subtract 8 from both sides
$-1 = c$ to find c then put both the gradient and the y intercept together as an equation

$y = 4x - 1$</td><td>Equation – letters, numbers and an equals sign.
Gradient – the slope of the line, found by doing $\frac{change\ in\ y}{change\ in\ x}$
y-intercept – where the line crosses the y axis

Remember that every coordinate is an x value and a y value (x, y)</td></tr>
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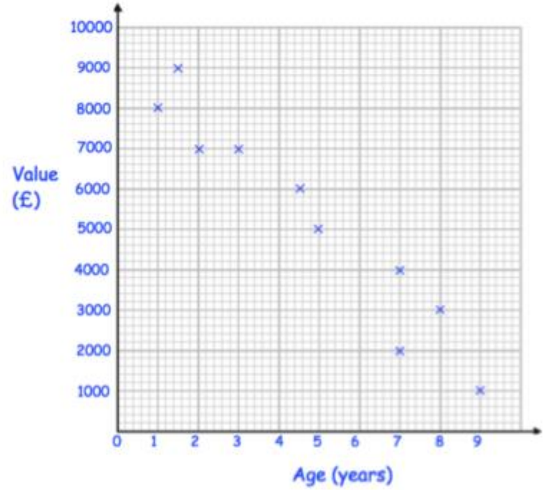
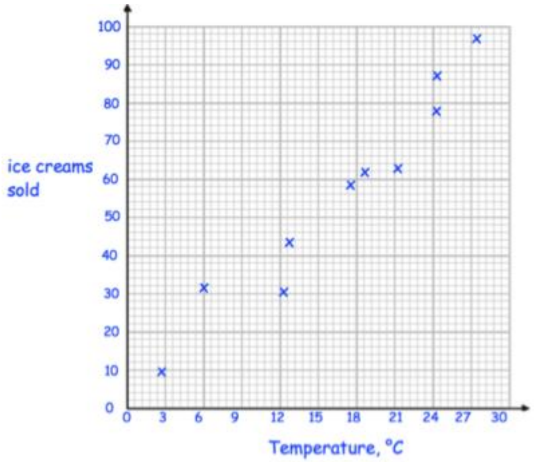
908	Solve a Pair of Linear Simultaneous Equations Example: Solve $5x + 3y = 22$ $2x + 4y = 20$ The method involves eliminating the letter in the middle of each equation (the y in this case) Multiply the top equation by the coefficient of the y on the bottom and vice versa $5x + 3y = 22$ x 4 $2x + 4y = 20$ x 3 This will give a 12y in both equations, but remember to multiply the entire equation each time $20x + 12y = 88$ $6x + 12y = 60$ Now both equations have a +12y in them we can subtract them to eliminate the ys $14x = 28$ And solve to find the value of x $x = 2$ We now know every x in the first equations was actually a 2, so sub this into one of them Using the first equation at the top $5(2) + 3y = 22$ 5(2) means 5 x 2 so will give 10 $10 + 3y = 22$ subtract 10 from each side $3y = 12$ and solve $y = 4$ So, our answer is x=2 and y=4	Simultaneous Equations are equations with two letters in each that have a single set of solutions, one for each letter. Linear means they do not contain any powers on the letters
909	Identify and Understand Positive and Negative Correlation Correlation is a link between two variables. It can be positive or negative. If it is positive it means that as one thing increase the other thing increases as well. If the correlation is negative it means that as one thing increases the other will decrease. IT is important to understand that positive correlation is not the same as causation. An increase in one thing will not ***cause*** the other thing to change, just that from observation if one thing is bigger the other things usually is as well. We can spot correlation on scatter graphs:	A variable is something we can measure. A scatter graph shows two variables at once on a graph. Each cross represents two values, one for each variable



Here as age increases, value decreases so
This is negative correlation

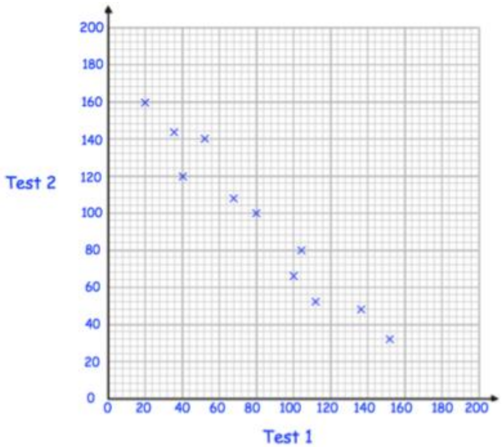


Here as temperature increase, ice creams sold also increases so this is positive correlation

910 Draw a Line of Best Fit and Use to Estimate from a Scatter Graph

A line of best fit shows the general trend on the scatter graph. It must be a straight line and it should have as many crosses above it as below it.

Example:



Jamie scored 60 on test one but missed test 2. By drawing a line of best fit, estimate his score for test 2

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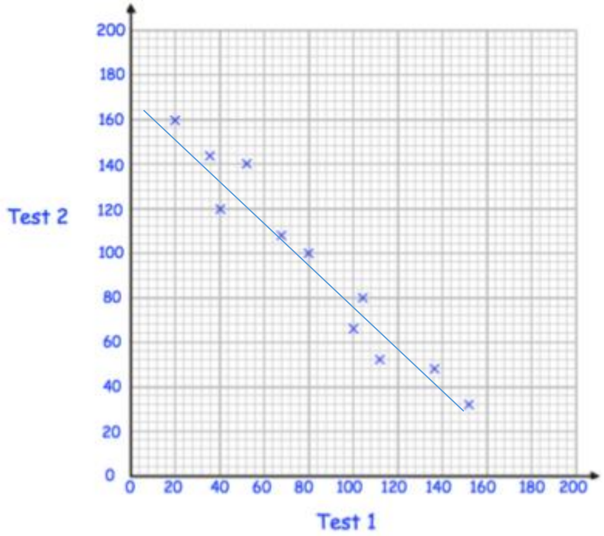
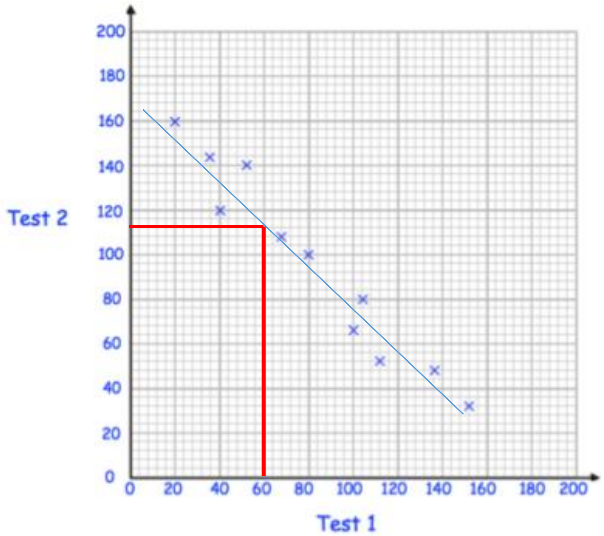


This shows generally where the line of best fit should be.



Go up from 60 on the test 1 axis (bottom) to the graph then go across to get an estimate of around 116 for Jamie’s test 2 score.

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Plot a Quadratic Graph

Use a table of values to substitute the given x values into your equation to get your y values and from that the coordinates that you will need to plot.

Example: Plot the graph of $y = x^2 + 2x - 5$ for values of x such that $-3 \leq x \leq 3$

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Quadratic – something containing an x^2
Substitute – replace letters with given numbers.

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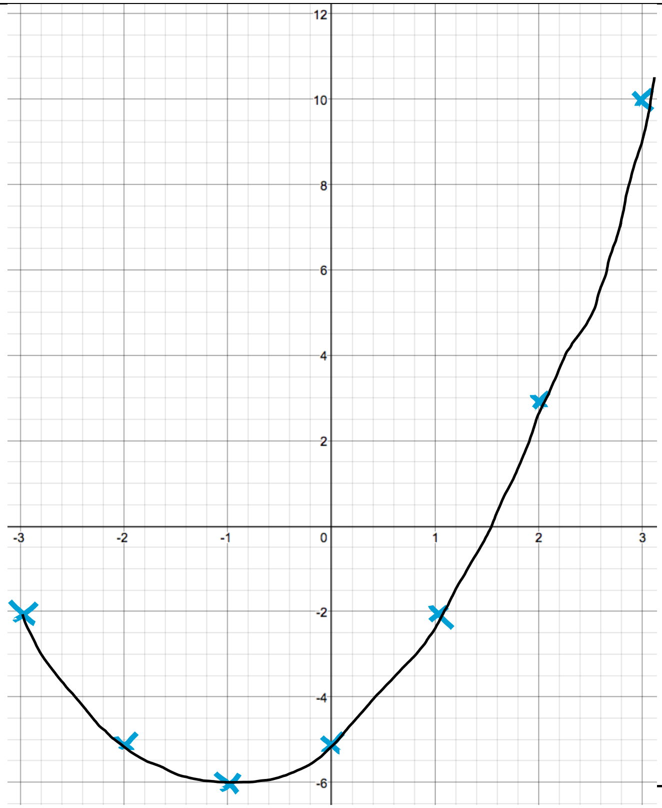
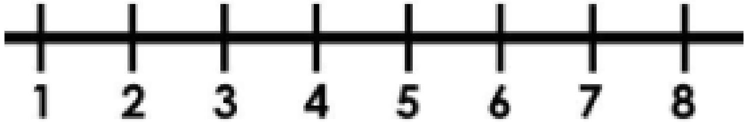
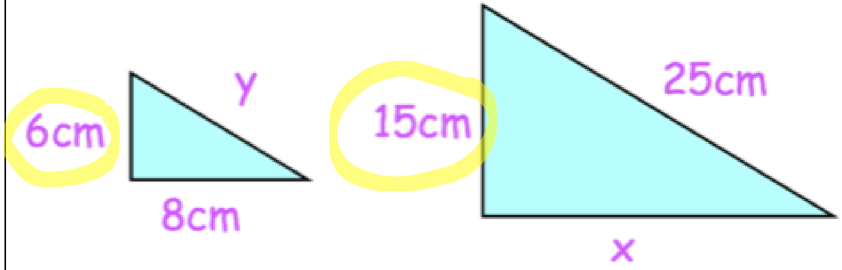
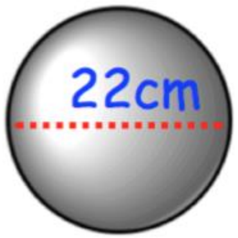
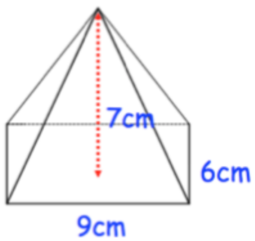
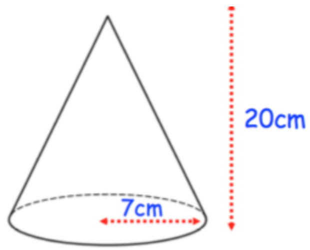
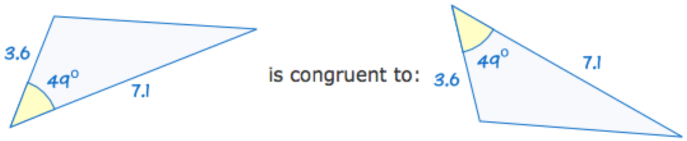
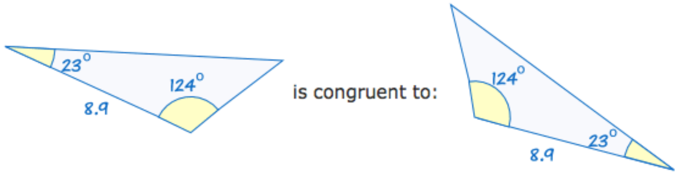
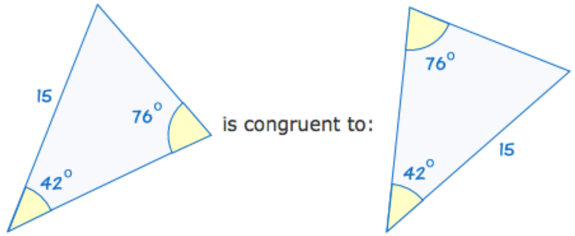
	This means that we will sub in all the whole numbers from -3 up to 3 ($-3 \leq x \leq 3$)into the equation $y = x^2 + 2x - 5$ and then plot the coordinates this gives us on a graph and join the dots up to form a smooth curve. Split the equation up into its terms x^2, $+2x$ and -5 and give each one a row in the table, putting the x values (-3 to 3) along the top of the table. Put y right at the bottom row as this is the other parts added together and will give us our number for the coordinate each time: <table><tr><td>x</td><td>-3</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>x^2</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr><tr><td>$+2x$</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr><tr><td>-5</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr><tr><td>y</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr></table>So we will now fill in each row by subbing the number on the top row into the expression down the left hand side. For example $(-3)^2$ $(-2)^2$ $(-1)^2$ etc for the top row. Careful with your negatives here. As the -5 does not have an x on it, it will always be -5 (we call this a constant term, the others are variable terms as they can change depending on what x is.) <table><tr><td>x</td><td>-3</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>x^2</td><td>9</td><td>4</td><td>1</td><td>0</td><td>1</td><td>4</td><td>9</td></tr><tr><td>$+2x$</td><td>-6</td><td>-4</td><td>-2</td><td>0</td><td>2</td><td>4</td><td>6</td></tr><tr><td>-5</td><td>-5</td><td>-5</td><td>-5</td><td>-5</td><td>-5</td><td>-5</td><td>-5</td></tr><tr><td>y</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr></table>Once each row is filled in, add the three parts of the equation together. So the first column gives us 9-6-5=-2 the second gives 4-4-5=-5 etc and write each answer in the y row along the bottom. <table><tr><td>x</td><td>-3</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>x^2</td><td>9</td><td>4</td><td>1</td><td>0</td><td>1</td><td>4</td><td>9</td></tr><tr><td>$+2x$</td><td>-6</td><td>-4</td><td>-2</td><td>0</td><td>2</td><td>4</td><td>6</td></tr><tr><td>-5</td><td>-5</td><td>-5</td><td>-5</td><td>-5</td><td>-5</td><td>-5</td><td>-5</td></tr><tr><td>y</td><td>-2</td><td>-5</td><td>-6</td><td>-5</td><td>-2</td><td>3</td><td>10</td></tr></table>Take each pair of x and y numbers from the top and bottom of the columns and these give you a coordinate (7 coordinates in total) $(-3,-2)$ $(-2,-5)$ $(-1,-6)$ $(0,-5)$ $(1,-2)$ $(2,3)$ $(3,10)$ and we now plot these on our graph	Note: quadratic graphs will form a parabola. Parabolas are shaped like smiles or frowns and will be symmetrical.

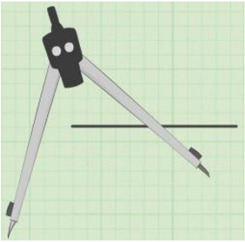
	 The crosses here are the coordinates that we worked out, so once they are plotted join the curve up as smoothly as you can.	
912	Solve an Inequality and Show it on a Number Line Solving an inequality uses the same inverse operations balancing method we use to solve equations then show this on a number line using circles and arrows to represent the inequality. Example: Solve the following inequality and represent it on a number line $5x + 9 > 29$ To solve this we'll need to -9 then divide by 5 -9 -9 $5x \quad > 20$ $\div 5 \quad \div 5$ $x > 4$ Our solution to the inequality is that *'x is greater than 4'* So on our number line put a blank circle above the 4 and have an arrow pointing towards the bigger numbers 	Inequality – has letters, numbers and a ≤<≥> sign < and > are shown by blank circles on the number line, whereas ≤and≥ are shown by coloured in circles

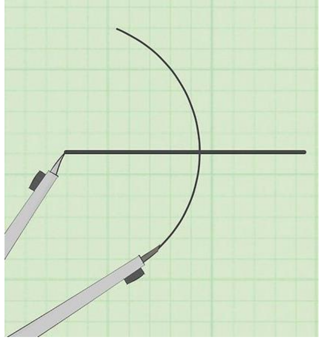
		
913	Solve a 'Work Done' Problem Work done problems require you to think using both direct proportion and indirect proportion. There are always 3 variables: things doing the work (people); time taken for the work to be done (time) and the amount of work to be done (work). No matter what order they appear in the question, set them out: **People Time Work** People and time are indirectly proportional (the more people working on a job, the shorter the time taken to complete it), but time and work are directly proportional (if the amount of work increases it will take longer to do it) Example: It takes 6 hours for 4 workers to unload 10kg of cargo. How long will it take 8 workers to unload 30 kg of cargo? People Time Work 4 6 10 Ignore the 10kg for the time being and work on getting the 'people' column to 8: divide the 4 1 24 10 By 4 to get to 1, which means the 6 will be multiplied by 4 (indirectly proportional) then 8 3 10 Multiply the people by 8 and divide the time by 8. This shows we know that 8 workers will take 3 hours to unload 10kg, and we want to know how long it will take them to unload 30kg so we can now only work with the second 2 columns and ignore the first 'people' column. 8 3 10 Time and Work are directly proportional so if we want to get from 10kg of work to 30kg of 8 9 30 Work we'll need to multiply by 3, so the 3 hours will also be multiplied by 3 Therefore, our answer is it will take 9 hours.	Direct proportion – an increase (multiplying) on one thing will cause the same increase (multiplying in the other eg. If one doubles the other one doubles. Indirect proportions – an increase in one thing means an decrease in the other by the same factor eg. If one thing is divided by 4 the other thing will be multiplied by 4
914	Use Similarity to Find Missing Sides on Shapes If shapes are similar in maths, one is the enlargement of the other. So, find the scale factor and use this to find the missing side.	Similar shapes are enlargements of each other. Scale factor – what you multiply the

	Example: The following shapes are similar, find the missing sides x and y  We have a pair of sides that are in the same place on the triangles, so doing 15cm÷6cm will give us the scale factor $15 \div 6 = 2.5$ The side x is in the same place that the 8cm is on the ***smaller*** triangle so doing $8 \times 2.5 = 20cm$ shows that x=20cm The side y is in the same place as the 25cm on the ***larger*** triangle so doing $25 \div 2.5 = 10cm$ shows that y=10cm	sides by to enlarge them. Once you've found the scale factor multiply to get sides on the larger shape and divide by the scale factor to get sides on the smaller shape.
915	Calculate the Volume of a Sphere, Pyramid or Cone These formulae are given on the exam paper, but it would be useful to memorise them. All you need to be able to do here really is put these into your calculator! $\boxed{\text{Volume of a sphere} = \frac{4}{3}\pi r^3 \quad \text{Volume of a pyramid} = \frac{1}{3} \times length \times width \times height \quad \text{Volume of a cone} = \frac{1}{3}\pi r^2 h}$ Notice that both the pyramid and cone formulae are effectively $\frac{1}{3}$ x the area of the base x height. This is because a cone is basically a circular based pyramid.    The 22cm here is the diameter Which means the radius will be 11cm:	None of these shapes are prisms as they do not have the same cross section (shape) running through them all.

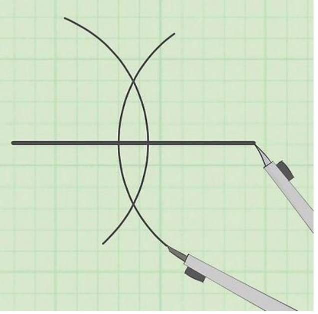
	Volume= $\frac{4}{3} \times \pi(11^3)$ =5575.3cm^3 Volume= $\frac{1}{3} \times 9 \times 6 \times 7$ =126cm^3 Volume= $\frac{1}{3} \times \pi(7^2)(20)$ =1026.3cm^3	
916	Identify and Continue a Fibonacci Sequence A Fibonacci sequence is where you find the next term by adding the previous two terms together. The famous Fibonacci sequence starts: 1, 1, 2, 3, 5, 8, 13, 21….etc Example: Find the next two terms in this sequence 3, 6, 9, 15, 24, 39 We can see it's a Fibonacci as we get each next term by adding the previous two terms. To get the next term we need to add together the final two terms given in the question: 24+39=63 then the next term after that will be 39+63=102 So, the next two terms are 63 and 102	A Fibonacci sequence is where the next term is found by adding the previous two terms
917	Identify and Continue/Generate a Quadratic Sequence Quadratic sequences only have a common difference (rule) when you look at the second differences For example: 2, 5, 10, 17, 26 has first differences 3, 5, 7 and 9, but second differences of 2 each time, this makes it quadratic 3 5 7 9 2 2 2 You can continue this sequence by spotting this patter and using it to get the next term: then next first difference would be 11 so add 11 to 26 and it gives you 37 as the next term. Generating a Quadratic from its n^{th}term Example: give the first 4 terms of the sequence $n^2 + 5$ Remember that n is the one times table $n = 1, 2, 3, 4$ so $n^2 = 1, 4, 9, 16$ adding 5 to each of these will give us our sequence $n^2 + 5 = 6, 9, 14, 20$	Quadratic means it involves an n^2 and its won't have the same term to term rule each time. Frist differences – the difference between each term in the sequence. Second difference – the differences between the first differences. n^{th} term the formula linking a terms position to its value
918	Know the Rules for Proving Congruency There are five rules that prove congruency. All involve abbreviations S=side A=angle	Congruent shapes are identical to each other.

	SSS – If all the sides are the same on both triangles, they are congruent  SAS – If two sides and ***the angle between*** them are the same, the triangles are congruent  ASA – If two angles and ***the side between them*** are the same, the triangles are congruent  AAS – if two angles are the same and another side is equal, the triangles are congruent  RHS – If it's a right-angles triangle and the hypotenuse and one other side are the same, the triangles are congruent  	Hypotenuse – largest side on a right-angled triangle.
919	Add and Multiply Column Vectors	A vector is something that has size and a direction for example

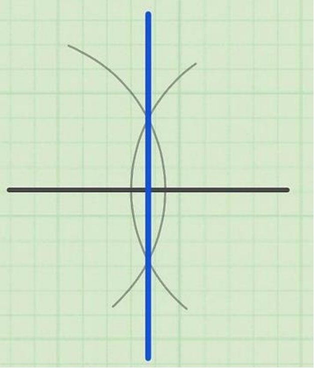
	The top number in a column vector tells you how many to go left (-) or right (+) and the bottom numbers tells you how many up (+) or down (-) to go so $\begin{Bmatrix} 3 \\ -5 \end{Bmatrix}$ means go 3 right and 5 down. To add column vectors just add the top numbers and add the bottom numbers, to multiply a vector multiply both numbers by the scalar. Example: Given the vectors $a = \begin{Bmatrix} 2 \\ 1 \end{Bmatrix}$ and $b = \begin{Bmatrix} -3 \\ 6 \end{Bmatrix}$ Calculate 2a+3b Substitute the given vectors into the expression here and work out your answer as a column vector $2\begin{Bmatrix} 2 \\ 1 \end{Bmatrix} + 3\begin{Bmatrix} -3 \\ 6 \end{Bmatrix}$ Multiply the first vector by its scalar or 2 and the second by its scalar of 3 $\begin{Bmatrix} 4 \\ 2 \end{Bmatrix} + \begin{Bmatrix} -9 \\ 18 \end{Bmatrix}$ Then add them by adding the top numbers and adding the bottom numbers $\begin{Bmatrix} -5 \\ 20 \end{Bmatrix}$	‘2 miles up’ is a vector, ‘west’ is not a vector. A scalar is a quantity used to ‘scale up’ or increase a vector: ‘3 right, 4 up’ with a scalar of 2 would give ‘6 right, 8 up’ Substitute means replace letters with numbers (in this case vectors) Expression is something that has letters and numbers but no equals sign
920	Construct Angle and Line Bisectors To Bisect a Line: Draw the line, if you haven’t been given it. Set the compass to more than half the length of the line.  Put the point on one end of the line and draw an arc from that point.	Construct means draw accurately using a compass and ruler. Bisect means to cut perfectly in half. An arc is part of a circle, you can draw full circles here if you want, but it complicates the diagrams a little bit if you do.



Put the point in the other end and draw an arc from that point, crossing the first arc in two places.



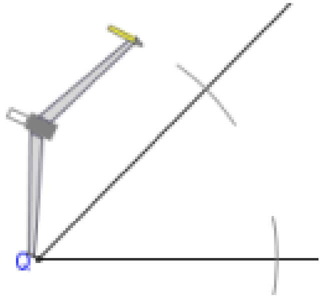
Join where the arcs cross



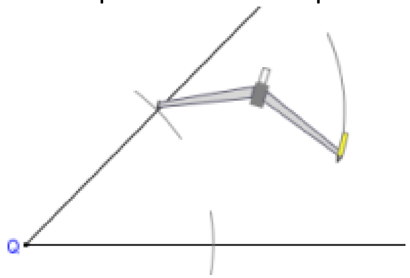
Angle Bisector

Draw the angle if you haven't been given it.

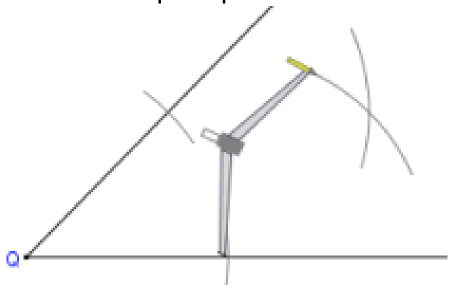
Put the point of your compass on the corner of the angle and draw an arc going through each line.



Put the point of the compass on where the arc has crossed one of the lines and draw an arc from there.



Put the compass point on the other place where the first arc crossed the other line and repeat.



Join the corner of the angle to where the second two arcs cross.